



# Triquark correlations and pentaquarks in a QCD sum rule approach

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## Abstract

The role of quark correlations in the description of hadron dynamics in many domains of physics, from low energy dynamics to very hot (dense) systems, is being appreciated. Strong correlations of two quarks (diquark) have been widely investigated in this respect. Recently, we have proposed a dynamical scheme to describe the  $\Theta^+$  pentaquark in which also three quark correlations (triquark) were instrumental in producing a low mass exotic state. We perform a study, within the QCD sum rule approach including OPE and direct instanton contributions, of triquark correlations and obtain two quasi-bound light  $ud\bar{s}$  color quark clusters of 800 and 930 MeV, respectively.

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## 1. Introduction

In 2003 evidence was reported of a very narrow exotic baryon of mass  $\approx 1540$  MeV and small width [1]. This so-called  $\Theta^+$  pentaquark with a minimal quark content  $uudd\bar{s}$  has motivated tremendous experimental and theoretical activity since its first sightings. Many experiments confirmed the observation [2] and new exotics were reported [3]. Theorists have aimed at understanding these states from the point of view of known low energy realizations of QCD [4,6–10]. With time the situation has become confusing [11]. The experimental status of  $\Theta^+$  is controversial since several experiments have reported searches with negative results and moreover no single experiment has confirmed the heavier exotics [12]. What seems to be established is that if these states exist they probe special features of QCD dynamics which will explain their rarity. Even if they do not exist, the efforts thus far have

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discovered dynamical features of QCD which favor clustering and which might be useful at higher densities and/or temperature.

The pentaquark arises quite naturally in chiral soliton schemes<sup>1</sup> [4]. However if one uses quark degrees of freedom the multiparticle nature of the state makes the dynamical analysis more elaborate. Conventional dynamics [6] leads to exotic baryons which are too heavy and their widths too large. Cluster schemes have been proposed which tend to explain the data. Jaffe and Wilczek rely on strong diquark correlations and Pauli blocking to generate a low mass, small width, state [7]. Lipkin and Karliner propose a triquark–diquark system induced by a generalized color magnetic interaction [8]. In our scheme [9], with the clustering of Karliner and Lipkin, the one gluon exchange (OGE) interaction plays a minor role and the correlations are built in by a strong instanton induced interaction (I3) [13,14]. The specific feature of the I3 leading to clustering is its strong flavor and spin dependence, i.e., due to the Pauli principle of the quarks in the zero modes of the instanton field the interaction is only nonvanishing between different quark flavors. The strength of the instanton induced attraction in the scalar–isoscalar diquark channel is enough to produce an almost bound color state [13].

In Ref. [9] we presented arguments in favor of the formation inside the pentaquark, due to the coupling of the instanton field, of a light color cluster with flavor content  $ud\bar{s}$ . In such a system a strong attraction is possible not only in the quark–quark, but also in quark–antiquark channel. Therefore, the feasibility for formation of a light,  $\approx 750$  MeV, triquark state was discussed. In order to confirm, from a more fundamental point of view, the results of our model calculation we proceed to use the QCD sum rule (SR) approach. As emphasized, in our model calculation, due to the particular spin-flavor-color structure of the pentaquark wave function, new types of two- and three-body I3s between the quarks, different from those appearing in conventional hadrons, are possible. Therefore, the analysis of the instanton effects on the properties of the multiquark hadrons within the SR approach is an interesting and actual problem, which describes new types of quark–quark correlations.

We present the first calculation of instanton effects in the multiquark sector of QCD within the QCD sum rule approach. Our considerations will start from the discussion of the direct instanton contribution to the sum rules for the nucleon and thereafter we will calculate instanton effects on the mass of a colored  $ud\bar{s}$  triquark.

## 2. QCD sum rule approach for the nucleon and triquark states

The study of correlations using SR is not new. A  $ud$ -diquark color system was considered within the QCD sum rule approach including instanton contributions [15,16] and it was shown that instanton induced attraction leads to a bound state for the isoscalar diquark with mass  $m_{ud} \approx 420\text{--}450$  MeV. Our model study [9] of the  $\Theta^+$  has discovered the possibility of physically relevant triquark correlations. Never mind the existence of the pentaquark, put in jeopardy by the last experimental analysis, the study of all kinds of quark correlations is an interesting project in itself, because they may be important dynamical mechanisms in various domains of physics.

One important conceptual distinction between the study of physical hadrons and color correlations using sum rules is the fact that the latter are not color singlets, and therefore are not physical states. The way to proceed is to build a color singlet current adding a sterile quark (antiquark) to them. In particular, a problem that has been discussed in detail is gauge independence in the extraction of their masses from the SR [15]. There, it is argued that one can consider colorless currents for the diquark (triquark) with an additional heavy quark (antiquark) and in this way avoid the problem of gauge invariance. However, since the heavy quarks (antiquarks) interact very weakly with the instantons, our results below will not change significantly.

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<sup>1</sup> Some authors have questioned the consistency of this calculation [5].

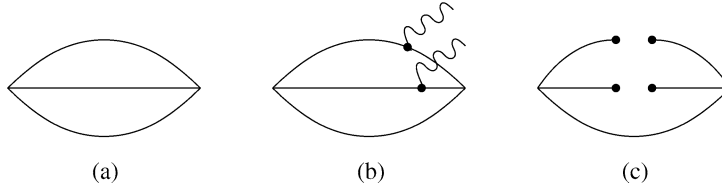


Fig. 1. Diagrams entering the calculation of  $\Pi_1(p^2)$  for the nucleon in the QCD sum rule OPE approach.

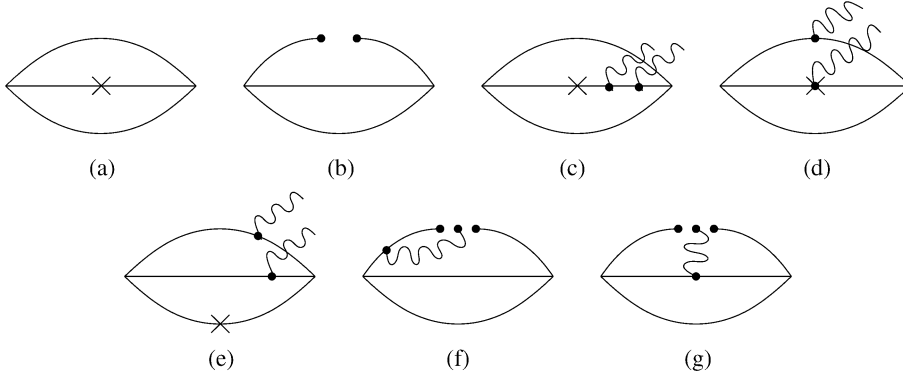


Fig. 2. Diagrams entering the calculation of  $\Pi_2(p^2)$  in the SR OPE approach. The insertion  $\times$  denotes the quark mass.

The main object in the SR approach, based on operator product expansion (OPE), is the correlator of two interpolating currents, with the quantum numbers of the particle under scrutiny, and which is given by

$$\Pi(p^2) = i \int d^4x e^{ip \cdot x} \langle 0 | T \eta(x) \bar{\eta}(0) | 0 \rangle. \quad (1)$$

For spin 1/2 baryons, the correlator can be decomposed into two functions

$$\Pi(p^2) = \hat{p} \Pi_1(p^2) + \Pi_2(p^2). \quad (2)$$

The spectral representation for the imaginary part of the correlator entering the dispersion relation is of the form

$$\text{Im } \Pi(s) = \pi |\lambda_B|^2 (\hat{p} + M_B) \delta(s^2 - M_B^2) + \pi \theta(s^2 - s_0^2) (\hat{p} \text{Im } \Pi_1(s^2) + \text{Im } \Pi_2(s^2)), \quad (3)$$

where  $M_B$  and  $\lambda_B$  are the mass and coupling strength of the ground state onto which the current projects, and  $s_0$  is a threshold, which will be used below to relate the properties of the nucleon and color triquark state with the OPE and the direct instanton contributions to the correlator.

We will consider the OPE contributions to the correlator up to the dimension six. The diagrams shown in Fig. 1 represent the OPE contributions to  $\Pi_1$  for the nucleon. The diagrams in Fig. 2 contain contributions for nonvanishing quark mass to  $\Pi_2$  and therefore can be applied to the nucleon and to the triquark state. We will only use  $\Pi_2$  to calculate the properties of the triquark state.

The quark propagator to this order, has the form

$$\begin{aligned} S_{ab}^q(x) &= -i \langle 0 | T q_a(x) \bar{q}_b(0) | 0 \rangle \\ &= \delta_{ab} (\hat{x} F_1^q + F_2^q) - i \tilde{g} G_{ab}^{\mu\nu} \frac{1}{x^2} (\hat{x} \sigma_{\mu\nu} + \sigma_{\mu\nu} \hat{x}) - m_q \tilde{g} G_{ab}^{\mu\nu} \sigma_{\mu\nu} \ln(-x^2), \end{aligned} \quad (4)$$

where  $a, b$  are the color indices and  $\tilde{g} = g_c/32\pi^2$ . The two functions entering the propagator are given by

$$F_1^q = \frac{1}{2\pi^2 x^4} + \frac{m_q \langle \bar{q}q \rangle}{48} + i \frac{m_q x^2}{27 \cdot 32} g_c \langle \bar{q} \sigma \cdot G q \rangle,$$

$$F_2^q = i \frac{m_q}{4\pi^2 x^2} + i \frac{\langle \bar{q}q \rangle}{12} - \frac{x^2}{192} g_c \langle \bar{q}\sigma \cdot Gq \rangle + i \frac{m_q g_c^2}{2^9 \cdot 3\pi^2} \langle G^2 \rangle x^2 \ln(-x^2), \quad (5)$$

where  $\langle \mathcal{O} \rangle$  denotes the vacuum condensate of the operator.

The gluon condensate and the mixed condensate are defined by

$$\langle G^2 \rangle = \langle G_{\mu\nu} G^{\mu\nu} \rangle, \quad \langle \bar{q}\sigma \cdot Gq \rangle = \langle \bar{q}\sigma_{\mu\nu} \cdot G^{\mu\nu} q \rangle \quad (6)$$

where  $\sigma_{\mu\nu}$  is defined by

$$\sigma_{\mu\nu} = \frac{1}{2} [\gamma_\mu, \gamma_\nu]. \quad (7)$$

We will assume that the current masses for the  $u, d$  quarks are zero, and therefore  $F_1^u = F_1^d, F_2^u = F_2^d$ .

Let us begin by discussing the nucleon. The positive parity interpolating current is

$$\eta_{\text{tot}}(x) = f\eta(x) + t\eta_1(x), \quad (8)$$

where  $f$  and  $t$  are two real parameters characterizing the mixing between the currents which are

$$\eta^N(x) = \epsilon^{abc} [u_a^T(x) \Gamma d_b(x)] u_c(x), \quad \eta_1^N(x) = \epsilon^{abc} [u_a^T(x) \Gamma_1 d_b(x)] \gamma_5 u_c(x), \quad (9)$$

with  $\Gamma = C\gamma_5, \Gamma_1 = C$ .

Using the conventional SR formalism after performing the Borel transform, we obtain two OPE sum rules for nucleon,

$$\frac{1}{4} (5t^2 + 2tf + 5f^2) E_2 M^6 + \frac{1}{16} (5t^2 + 2tf + 5f^2) b E_0 M^2 + \frac{2}{3} (7f^2 - 2tf - 5t^2) a^2 = \tilde{\lambda}_N^2 e^{-M_N^2/M^2}, \quad (10)$$

$$(7f^2 - 2tf - 5t^2) a E_1 M^4 - 3(f^2 - t^2) m_0^2 a E_0 M^2 = \tilde{\lambda}_N^2 M_N e^{-M_N^2/M^2}. \quad (11)$$

Let us apply the same formalism to the  $ud\bar{s}$  triquark state with isospin  $I = 0$ , spin  $S = 1/2$  and color  $C = 3_c$  in the pentaquark. We will consider two types of triquark states with different color structure for the  $ud$  subsystem labeled  $A, C_{ud} = \bar{3}_c$  and  $B, C_{ud} = 6_c$  (see [9]).

The  $A$  state has a nonvanishing overlap with the currents

$$\eta^A = \frac{1}{4} \epsilon_{abc} \epsilon_{bde} [u_d^T \Gamma d_e] C \bar{s}_c^T, \quad \eta_1^A = \frac{1}{4} \epsilon_{abc} \epsilon_{bde} [u_d^T \Gamma_1 d_e] \gamma_5 C \bar{s}_c^T, \quad (12)$$

which correspond to the mixture of scalar and pseudoscalar isosinglet ( $ud$ ) diquark appearing in its wave function.

The  $B$  state has a nonvanishing overlap with the current<sup>2</sup>

$$\eta^B(x) = \frac{1}{4\sqrt{3}} [u_a^T(x) C \gamma_\mu d_b(x) + u_b^T(x) C \gamma_\mu d_a(x)] \gamma_5 \gamma^\mu C \bar{s}_b^T. \quad (13)$$

All these interpolating currents are of negative parity.

For the triquark states only chirality odd SR will be considered, because, as will be shown later, only chirality odd SR have a good stability plateau when direct instanton contributions are incorporated.

Thus, for the  $A$  state we have

$$(f^2 - t^2) \left( \frac{m_s}{6} E_2 M^6 + \frac{f_s}{6} a E_1 M^4 - \frac{f_s}{12} a m_0^2 E_0 M^2 - \frac{m_s}{24} b E_0 M^2 (v(M^2) - 1/2) \right) = \tilde{\lambda}_A^2 M_A e^{-M_A^2/M^2}, \quad (14)$$

<sup>2</sup> There is another current defined in terms of the tensor  $\sigma_{\mu\nu}$  which is not relevant for the purposes of the present calculation.

and for the  $B$  state the result is

$$\frac{2}{9}m_s E_2 M^6 + \frac{2}{9}f_s a E_1 M^4 + \frac{1}{36}f_s m_0^2 a E_0 M^2 + \frac{1}{72}m_s b E_0 M^2 (v(M) + 1) = \tilde{\lambda}_B^2 M_B e^{-M_B^2/M^2}, \quad (15)$$

where  $v(M) = \ln(M^2 \rho_c^2/4) + \gamma_{EM} - 1$ .

Above we considered contributions to the correlator up to dimension six and up to orders  $\mathcal{O}(m_s)$ ,  $\mathcal{O}(m_s g_c)$ ,  $\mathcal{O}(m_s g_c^2)$ , and did not consider  $\mathcal{O}(\alpha_s)$  corrections. The parameters are introduced by the following relations

$$\begin{aligned} \langle \bar{u}u \rangle &= -\frac{a}{(2\pi)^2}, & g_c^2 \langle G \cdot G \rangle &= b, & i g_c \langle \bar{u}\sigma \cdot Gu \rangle &= m_0^2 \langle \bar{u}u \rangle, \\ \frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle} &= \frac{\langle \bar{s}\sigma \cdot Gs \rangle}{\langle \bar{u}\sigma \cdot Gu \rangle} = f_s, & \tilde{\lambda}_B &= (4\pi)^2 \lambda_B, \end{aligned} \quad (16)$$

and the needed functions  $E_n$  are given by

$$E_n(x) = 1 - e^{-x} \sum_n \frac{x^n}{n!}, \quad \text{with } x = \frac{s_0^2}{M_B^2}. \quad (17)$$

We will use the following values for parameters [17]

$$\langle \bar{q}q \rangle = -(250 \text{ MeV})^3, \quad b = 0.24 \text{ GeV}^4, \quad m_0^2 = 1 \text{ GeV}^2, \quad m_s = 150 \text{ MeV}, \quad f_s = 0.8.$$

### 3. QCD sum rules with direct instanton contributions

In the OPE based SR for the nucleon, the contributions due to large size vacuum fluctuations of quark and gluon fields have been taken into account (Figs. 1 and 2). If only such fluctuations are important in determining the mass of a particle, with given quantum numbers, then the OPE is valid and one can safely restrict the calculation to a finite number of terms in the expansion. However, in the QCD vacuum, there are strong fluctuations of small size associated with the gluon fields, namely the instantons, which can lead to a significant modification of the OPE QCD sum rules [18–21]. For example, the instantons can produce a large violation of factorization in some four-quark vacuum–vacuum matrix elements and can lead to the appearance of additional exponential terms in the current correlator which have nothing to do with the standard power-like  $1/q^{2n}$  of the OPE expansion.

We proceed to incorporate the instanton contributions in our calculation. To do so we have to rotate all our equations to Euclidean space–time, where the instantons are defined, according to  $\hat{x}_M = -i\hat{x}_E$ ,  $x_M^2 = -x_E^2$ , and  $\langle \bar{q}q \rangle_M = -i\langle \bar{q}q \rangle_E$ .

The propagator has two terms, the standard one (st) and the one associated to the instanton contributions (inst),

$$S_{ab}^q(x, y) = S_{ab}^{q,\text{st}}(x, y) + S_{ab}^{q,\text{inst}}(x, y). \quad (18)$$

For the standard quark propagator  $S^{q,\text{st}}$  we use the free propagator with mass and quark condensate corrections, i.e.,

$$\begin{aligned} S_{ab}^{q,\text{st}}(x, y) &= \delta_{ab} \left( \frac{\hat{x} - \hat{y}}{2\pi^2(x-y)^4} + i \frac{m_q}{4\pi^2(x-y)^2} + i \frac{\langle \bar{q}q \rangle}{12} \right) \\ &\rightarrow \delta_{ab} \left( -i \frac{(\hat{x} - \hat{y})_E}{2\pi^2(x-y)^4} - i \frac{m_q}{4\pi^2(x-y)^2} + \frac{\langle \bar{q}q \rangle_E}{12} \right). \end{aligned} \quad (19)$$

The leading effect of instantons is provided by the zero quark mode approximation which leads to the following ansatz for the quark propagator in the instanton background, [13]

$$S_{ab}^{q,\text{inst}}(x, y) = A_q(x, y) [(\hat{x} - \hat{z}_0) \gamma_\mu \gamma_\nu (\hat{y} - \hat{z}_0) (1 - \gamma_5)] (U \tau_\mu^- \tau_\nu^+ U^\dagger)_{ab}, \quad (20)$$

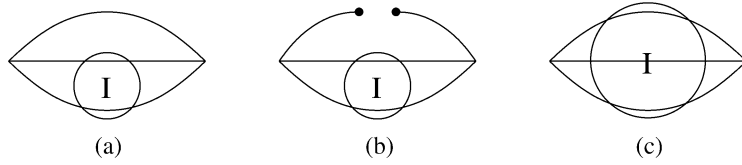


Fig. 3. The (a) two-body instanton induced contribution to  $\Pi_1$ , (b) two-body instanton contribution to  $\Pi_2$  and (c) three-body instanton contribution to  $\Pi_2$ . In the figure  $I$  denotes the instanton.

where

$$A_q(x, y) = -i \frac{\rho^2}{16\pi^2 m_q^*} \phi(x - z_0) \phi(y - z_0)$$

and

$$\phi(x - z_0) = \frac{1}{\sqrt{(x - z_0)^2 [(x - z_0)^2 + \rho^2]^{3/2}}}.$$

Note that  $\rho$  stands for the instanton size,  $\rho_c$  is the average instanton size and  $z_0$  the center of the instanton;  $U$  represents the color orientation matrix of the instanton in  $SU(3)_c$  and  $\tau_{\mu, \nu}$  are  $SU(2)_c$  matrices;  $m_q^* = m_{\text{cur}}^q - 2\pi^2 \rho_c^2 \langle \bar{q}q \rangle / 3$  is the effective quark mass in the instanton vacuum and  $m_{\text{cur}}^q$  the current quark mass. The final result should be multiplied by a factor of two, due to the antiinstanton contribution, and has to be integrated over the instanton density  $\int n(\rho) d\rho$ .<sup>3</sup>

An important selection rule for the quarks in the instanton field reads

$$\vec{\sigma}_i \oplus \vec{\tau}_i = 0, \quad (21)$$

where  $\sigma_i$  is usual spin and  $\tau_i$  is color spin of the quark. This selection rule leads to the vanishing of the instanton two-body quark contribution to masses of particles from the baryon decuplet and forbids also the three-body instanton induced interaction to the colorless baryons. In Fig. 3 the two-body and three-body instanton induced contributions to the current correlator are shown.

Using a model for instanton density defined by  $n(\rho) = n_{\text{eff}} \delta(\rho - \rho_c)$  [18], we calculate the instanton contributions to the correlator of the nucleon current, which after Borel transformation are given by

$$\Pi_1^N(M) = \frac{3n_{\text{eff}}(f^2 - t^2)}{4^3 \pi^2 \rho_c^4 (m_u^*)^2} \left[ \frac{64}{5} \left( 1 - \frac{24}{7z^2} \right) + \frac{z^8}{4} \int_0^1 dy \frac{1}{y^2(1-y)^2} \frac{(X^3 + 6X^2 + 18X + 24)e^{-X}}{X^5} \right], \quad (22)$$

$$\Pi_2^N(M) = -\frac{n_{\text{eff}} \langle \bar{q}q \rangle M \rho_c^4}{3 \cdot 2^6 (m_u^*)^2} (13t^2 + 10tf + 13f^2) M^6 e^{-z^2/2} (K_0(z^2/2) + K_1(z^2/2)), \quad (23)$$

where  $z = M\rho_c$  and  $X = z^2/(4y(1-y))$ .

The instanton contribution to the color  $ud\bar{s}$  states has a more complicated structure. The two-body instanton effects to  $\Pi_2$  for the correlator of state  $A$  is given, in configuration space, by

<sup>3</sup> For a discussion on the possible values of the parameters of this instanton model see Ref. [15].

$$\begin{aligned}
\langle T\eta_{\text{tot}}^A(x)\bar{\eta}_{\text{tot}}^A(0) \rangle^2 &= \frac{i n_{\text{eff}} \rho_c^4}{2\pi^4} \frac{1}{z_0^2(x-z_0)^2[(x-z_0)^2 + \rho_c^2]^3[z_0^2 + \rho_c^2]^3} \\
&\times \left\{ \frac{z_0^2(x-z_0)^2}{3m_u^*m_d^*} \left[ (t^2 + f^2) \left( \frac{\langle \bar{s}s \rangle_E}{12} - i \frac{m_s}{4\pi^2 x^2} \right) - (t^2 - f^2) \frac{i\hat{x}}{2\pi^2 x^4} \right] \right. \\
&\left. + (t+f)^2 \frac{\langle \bar{q}q \rangle_E}{4^3 \cdot 6m_u^*m_s^*} \left( 4((x-z_0) \cdot z_0)^2 - \frac{4}{3}((x \cdot z_0)^2 - x^2 z_0^2) \right) \right\}, \quad (24)
\end{aligned}$$

where we have used the assumption  $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle$  and denoted them by  $\langle \bar{q}q \rangle$ , while the three-body contribution is given by

$$\langle T\eta_{\text{tot}}^A(x)\bar{\eta}_{\text{tot}}^A(0) \rangle^3 = -(t+f)^2 \frac{n_{\text{eff}} \rho_c^6}{12\pi^6 m_u^* m_d^* m_s^*} \frac{(x-z_0)z_0}{\sqrt{(x-z_0)^2} \sqrt{z_0^2} [(x-z_0)^2 + \rho_c^2]^{9/2} [z_0^2 + \rho_c^2]^{9/2}}. \quad (25)$$

The two-body instanton effects to  $\Pi_2$  for the correlator of state  $B$  is given, in configuration space, by

$$\langle T\eta^B(x)\bar{\eta}^B(0) \rangle^2 = i \frac{11 n_{\text{eff}} \rho_c^4 \langle \bar{q}q \rangle_E}{4 \cdot 108\pi^4 m_q^* m_s^*} \frac{1}{[z_0^2 + \rho_c^2]^3 [(x-z_0)^2 + \rho_c^2]^3}, \quad (26)$$

while the three-body exactly vanishes.

The Borel transform of the correlator for  $A$  for arbitrary values of the parameters  $f$  and  $t$  has a rather complicated form and has to be calculated numerically. For the  $B$  state it is simple and the proportional to the two body nucleon one

$$\Pi_2^B(M)_{\text{inst}} = - \frac{11 n_{\text{eff}} \langle \bar{q}q \rangle_M \rho_c^4}{4 \cdot 1728 m_u^* m_s^*} M^6 e^{-z^2/2} (K_0(z^2/2) + K_1(z^2/2)). \quad (27)$$

Our estimates of instanton effects make use of the following relation between the parameters of the Shuryak instanton model [22]

$$\frac{2n_{\text{eff}}}{m_q^{*2}} = \frac{3}{2\pi^2 \rho_c^2}. \quad (28)$$

Furthermore, it turns out that, in the model, the size of the instanton contribution is determined only by value of the average instanton size in the QCD vacuum  $\rho_c$ .

Let us discuss first the instanton contribution to the nucleon case. In the literature there are two slightly different statements about the effects of instantons on the stability of the nucleon SR. In Ref. [20] it was shown that the instantons lead to a significant improvement of the stability of the chiral odd  $\Pi_2$  SR and do not improve the stability of chiral even  $\Pi_1$  SR. However, in Ref. [21] it was argued that the instanton contribution (22) also leads to the appearance of a stability plateau as a function of Borel mass in the chiral even SR for  $f \neq -t$ .

Our present exact calculation confirms the results of Ref. [20] and gives rise to a stability plateau for the chiral odd SR (Figs. 4 and 5). We also obtain the experimental mass of the nucleon,  $M_N = 940$  MeV, for a reasonable average size of the instantons,  $\rho_c \approx 1.6$  GeV<sup>-1</sup>. The existence of a stability plateau in  $\Pi_2$  SR is not very sensitive to the values of the parameters of the nucleon current  $f$  and  $t$ . However, we did not find such a stability plateau for the chiral even SR for any choice of the current parameters  $f$  and  $t$ . We should lastly mention that for the Ioffe current  $f = -t$ , the instanton contribution to chiral even SR vanishes explicitly (recall Eq. (22)). Therefore chiral even SR are not considered in our discussion below.

The study of the SR provides us with a very light negative parity  $A$  triquark state whose mass is  $M_{\text{tri}}^A \approx 800$  MeV for a mixed  $A$  type current with  $f \approx -t$ . This state shows a stability plateau as function of Borel parameter for both mass and residue (Figs. 6 and 7).  $\lambda_A^2$  is positive which insures that the parity is negative. For the  $B$  current we also find a negative parity state again with a stability plateau for both mass and residue (Figs. 8 and 9). The value of its mass is  $M_{\text{tri}}^B \approx 930$  MeV and again  $\lambda_B^2$  is positive.

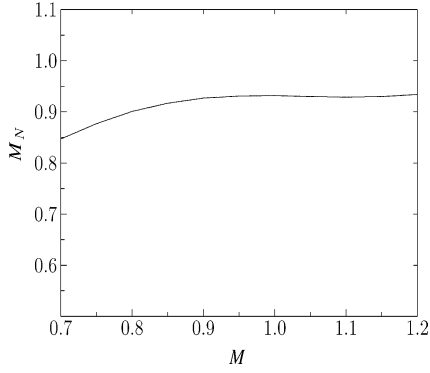


Fig. 4. The nucleon mass as a function of the Borel parameter  $M$  for the chiral odd SR with  $s_0 = 1.75$  GeV.

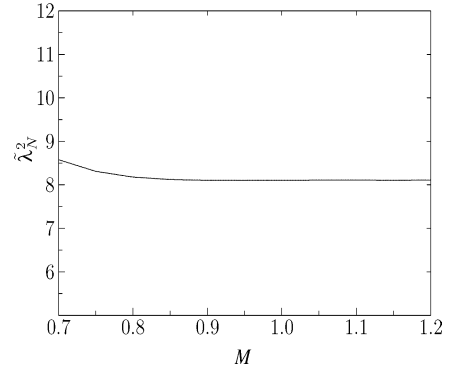


Fig. 5. The nucleon residue as a function of the Borel parameter  $M$  for chiral odd SR with  $s_0 = 1.75$  GeV.

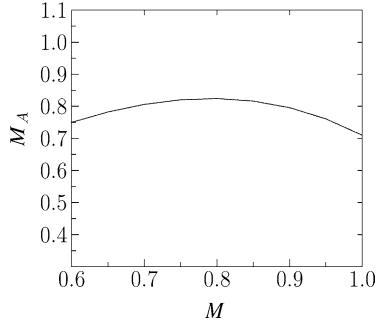


Fig. 6. The  $A$  state mass obtained incorporating the instanton contributions as a function of the Borel parameter for chiral odd SR with  $s_0 = 1.8$  GeV.

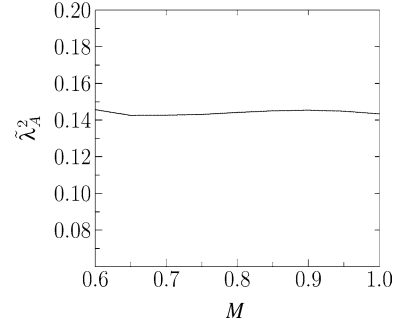


Fig. 7. The  $A$  state residue obtained including the instanton contributions as a function of Borel parameter for chiral odd SR with  $s_0 = 1.8$  GeV.

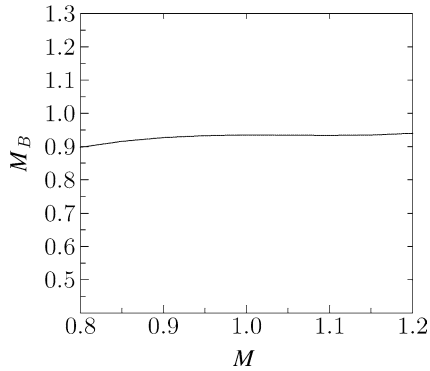


Fig. 8. The  $B$  state mass containing the instanton contributions as a function of Borel parameter  $M$  from chiral odd SR with  $s_0 = 1.72$  GeV.

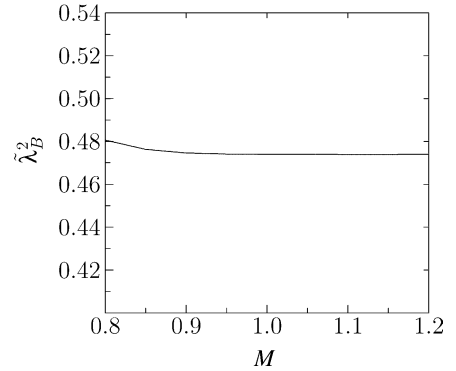


Fig. 9. The  $B$  state residue incorporating the instanton contributions as a function of Borel parameter  $M$  for chiral odd SR with  $s_0 = 1.72$  GeV.



It should be emphasized that without the contribution of the instantons our analysis of the chiral odd triquark SR would have shown an absence of stability plateau for both  $A$  and  $B$  states. Therefore their mass would have been difficult to determine.

Our calculation shows that three-body contribution for triquark  $A$  state is very small and vanishes for a Ioffe type triquark current ( $f = -t$ ) as well as for the  $B$  current. Therefore, we expect that three-body instanton induced forces do not play a significant role in multi-quark systems. This conclusion is in agreement with the result of the calculation of the three-body instanton contribution to the mass of the  $H$ -dibaryon within a bag model [23].

Recalling the investigations with diquarks [15,16] and at the light of our present results it becomes natural to consider a model for a light pentaquark as an  $A$ – $B$  (mixed) triquark–( $ud$ ) diquark system, with a coupling between the clusters with nonzero orbital momentum  $L = 1$  [9]. In this case centrifugal barrier will suppress quark rearrangement between the two color clusters. Furthermore, an additional orbital excitation energy  $\approx 400$  MeV (see [9]) will bring the total mass of pentaquark to its observed value. The heavier pentaquark  $B$ – $A$ (mixed)–triquark–( $ud$ ) diquark orthogonal system is expected to have a mass about 200 MeV higher. Due to negative internal parity of the light triquark state the total parity of pentaquark system in this case is positive in agreement with the expectation of the soliton model [4].

#### 4. Conclusion

The dynamics of correlated quarks is being appreciated in many areas of hadron physics. The “wishful” discovery of the  $\Theta^+$  and its immediate consequences on the spectrum would allow the study of multiparticle correlations in QCD in a natural scenario. In the undesirable circumstances that the pentaquarks, and other exotics, do not exist the study of quark correlations in other domains of hadron physics will open up the possibility of further understanding the dynamics of QCD. The aim of this presentation has been to single out the importance of the instantons in the multiparticle dynamics of QCD.

In order to do so we have incorporated in a traditional OPE calculation of SR the direct instanton effects for triquark  $ud\bar{s}$  color clusters. We have shown that instantons lead to a large stability for the correlator of the color triquark current as a function of the Borel parameter. We observe the formation of two negative parity  $ud\bar{s}$  states with spin one-half and isospin zero. These particular triquark states [9] might be behind the unusual properties of the observed pentaquark state and support the Karliner and Lipkin triquark–diquark clusterization scheme [8]. We emphasize that all published calculations of masses of pentaquark within QCD sum rules [17,24] should be reanalyzed including the direct instanton contributions.

We hope that our investigation inspires the study of quark correlations using lattice theory, a theoretical support to prove the existence of exotics, and the role played by the instantons in their dynamics, using the appropriate techniques [25]. Finally, it is clear that the  $\Theta^+$  has become now, above all, an experimental issue which will be solved in the near future, but our study indicates that correlations are a consequence of the way we understand QCD dynamics and we hope to inspire the search for other experimental scenarios where they might play an important role.

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